PRE-BOARD CLASS XII (MATHS) 15-1-18

Marking Scheem of $\boxed{\text{Set - B}}$

Sec A

4) $y = \sin^{-1}\sin(x-\pi) = x-\pi \in [-\pi/2, \pi/2)$ 1

$\boxed{\frac{dy}{dx} = 1}$ or, $x \in [\pi/2, 3\pi/2] \Rightarrow y = \sin^{-1}\sin(-x)$

$\boxed{\frac{dy}{dx} = -1}$

See-B

8) $x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$ 1

Sq. both side

$x^2(1+y) = y^2(1+x)$

$x^2 + x^2y = y^2 + y^2x$

$x^2 - y^2 = y^2x - x^2y$

$(x-y)(x+y) - xy(y-x) = 0$ 1

$(x-y) = 0$ or $x+y+xy = 0$

$x \neq y$ or $y(1+x) = -x$

or $y = \frac{-x}{1+x}$

$\frac{dy}{dx} = \frac{-1}{(1+x)^2}$.

9) Angle b/w plane & line,

$\sin\theta = \frac{2\cdot4 + 4\cdot5 + 5\cdot6}{\sqrt{4+16+25}\sqrt{16+25+36}}$ 1

$= \frac{58}{\sqrt{45}\sqrt{77}}$

$\Rightarrow \theta = \sin^{-1}\left(\frac{58}{\sqrt{45}\sqrt{77}}\right)$. 1

Sec-C

21) $\int\left(\sqrt{\tan x} + \sqrt{\cot x}\right)dx$

$= \int\left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}}\right)dx = \int \frac{\sin x + \cos x}{\sqrt{\sin x \cdot \cos x}}dx$ 1

$$= \int \frac{\sqrt{2}(\sin x + \cos x)}{\sqrt{1 + \sin 2x - 1}} dx$$

$$= \int \frac{\sqrt{2}(\sin x + \cos x)\, dx}{\sqrt{1 - (1 - \sin 2x)}}$$

$$= \int \frac{\sqrt{2}(\sin x + \cos x)\, dx}{\sqrt{1 - (\sin x - \cos x)^2}}$$

putting
$\sin x - \cos x = t$
$(\cos x + \sin x)dx = dt$

$$= \sqrt{2} \int \frac{dt}{\sqrt{1 - t^2}}$$

$$= \sqrt{2} \sin^{-1}(\sin x - \cos x) + c \quad \text{Ans}$$

OR

$$\int e^x \left( \frac{1 + \sin x}{1 + \cos x} \right) dx$$

$$= \int e^x \left( \frac{1 + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right) dx$$

$$= \int e^x \left( \frac{1}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right) dx$$

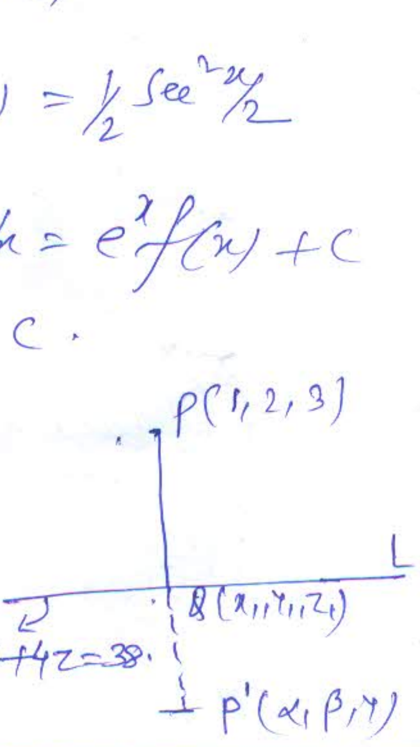
$$f(x) = \tan \frac{x}{2}, \quad f'(x) = \frac{1}{2} \sec^2 \frac{x}{2}$$

$$= \int e^x (f(x) + f'(x))\, dx = e^x f(x) + c$$

$$= e^x \tan \frac{x}{2} + c.$$

Sec-D

29) Let foot be $Q(x_1, y_1, z_1)$
& image be $P'(\alpha, \beta, \gamma)$.
& equn of plane is
$x + 2y + 4z = 38$ — ①



Sec-D



22) 1) Identity element in A

$(a,b) * (c,d) = (ac, b+ad)$

$\Rightarrow (a,b) * (e,f) = (ae, b+af) = (a,b)$

$\Rightarrow ae = a$ & $\cancel{b}+af = \cancel{b}$

$e = 1$ , $f = 0$

Identity elt. $(1,0)$

2) Invertible.

$(a,b) * (h,k) = (e,f)$

$\Rightarrow (ah, b+ak) = (1,0)$

$\Rightarrow ah = 1$ or $b+ak = 0$

$h = 1/a$ or $ak = -b$

$k = -b/a$

Invertible elts $(1/a, -b/a)$.

Now Direction ratio PQ is $(x_1-1, y_1-2, z_1-3)$

& Direction ratio of plane is $(1, 2, 4)$

Here Line PQ $\perp^r$ plane

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{x_1-1}{1} = \frac{y_1-2}{2} = \frac{z_1-3}{4} = K \text{ (Say)}$$

$$\Rightarrow x_1 = K+1,\ y_1 = 2K+2 \ \&\ z_1 = 4K+3.$$

$\because (x_1, y_1, z_1)$ lies on plane $x+2y+4z=38$

$$\therefore x_1 + 2y_1 + 4z_1 = 38$$

$$\Rightarrow K+1+2(2K+2)+4(4K+3) = 38$$

$$\Rightarrow K+1+4K+4+16K+12 = 38$$

$$\Rightarrow 21K = 21 \Rightarrow \boxed{K=1}$$

$\&\ x_1 = 2,\ y_1 = 4\ \&\ z_1 = 7$

foot $(2, 4, 7)$

$\because$ Q is the mid-pt. of PP'

$$\therefore \frac{\alpha+1}{2} = x_1,\ \frac{\beta+2}{2} = y_1\ \&\ \frac{\gamma+3}{2} = z_1$$

$$\Rightarrow \frac{\alpha+1}{2} = 2,\ \frac{\beta+2}{2} = 4\ \&\ \frac{\gamma+3}{2} = 7$$

$$\Rightarrow \alpha = 4-1,\ \beta = 4\cdot2-2 = 6\ \&\ \gamma = 14-3$$

$$\alpha = 3 \qquad \gamma = 11$$

& Image of point $(3, 6, 11)$

—x—

P.T.O