PRE-BOARD EXAM 2018

15-1-18 ①

CLASS - XII

SET-A

MATHEMATICS :- Marking Scheme. MAX. MARKS: 100

PREBOARD EXAM-(2017-2018)

Q. No.		Key Points
	Sec-A	
1)	$\vec{a} \parallel \vec{b} \Rightarrow \frac{3}{1} = \frac{2}{p} = \frac{9}{3} \Rightarrow p = \frac{2}{3}$.	1
2)	Area of Δ = 5 sq. unit $\Rightarrow \frac{1}{2}\begin{vmatrix} K & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 3 & 1 \end{vmatrix} = \pm 5$ $\Rightarrow -2k+3 = \pm 10$ $\Rightarrow -2k+3 = 10$ or $-2k+3 = -10$ $-2k = 7$ or $-2k = -13$ $k = -7/2$ $k = 13/2$.	1
3)	$\int_0^1 (3x^2+2x+k)\,dx = 0$ $\Rightarrow \left[\frac{\cancel{3}x^3}{\cancel{3}} + \frac{\cancel{2}x^2}{\cancel{2}} + kx\right]_0^1 = 0$ $\Rightarrow 1+1+k = 0 \Rightarrow \boxed{K=-2}$	1
4)	$\lvert adj A \rvert = \lvert A \rvert^{n-1} = (5)^2 = 25$	1
	Sec-B	
5)	The principal diagonal elements of a skew-symmetric matrix are always zero, because if we put $i=j$ in $a_{ji} = -a_{ij}$ then $a_{ii} = -a_{ii} \Rightarrow 2a_{ii} = 0 \Rightarrow a_{ii} = 0 \; \forall i$ $\Rightarrow$ So that Det A = 0.	1 1
6)	Rolle's theorem be satisfied $f'(c) = 0$ $\Rightarrow 3c^2 - 3 = 0$, $c = \pm 1 \in [-3, 0]$, So that $\boxed{c=-1}$.	1
7)	We have $\frac{d}{dt}x^3 = 9$.. $\Rightarrow 3x^2\frac{dx}{dt} = 9$ $\Rightarrow \frac{dx}{dt} = \frac{3}{x^2}$ cm/sec	1

Q.No. | Hints to the answer | Key

We have $S = 6x^2$

$$\frac{ds}{dt} = 12\frac{dx}{dt} = 12\left(\frac{3}{x^2}\right)$$ 1

$$\therefore \left.\frac{ds}{dt}\right]_{x=10} = \frac{36}{100} = 0.36\ cm^2/sec.$$

8) $f(x) = x^3 - 3x^2 + 6x - 100$ 1

$f'(x) = 3x^2 - 6x + 6$

$= 3(x^2 - 2x + 2)$ 1

$= 3(x^2 - 2x + 1 + 1)$

$= 3((x-1)^2 + 1) \geq 0$ i.e $(x-1)^2$ is always +ve.

so $f(x)$ is increasing on R.

9) When lines are $\perp^r$

$a_1a_2 + b_1b_2 + c_1c_2 = 0$ 1

$2.1 + 3.2 + 4.(-2) = 2 + 6 - 8 = 0.$ 1

Proved.

10) $n(S) = 6$

Let A : Number is even $= \{2, 4, 6\} \Rightarrow n(A) = 3$ ½

B : Number is red $= \{1, 2, 3\} \Rightarrow n(B) = 3$

$A \cap B = \{2\} \Rightarrow n(A \cap B) = 1$

$P(A) = \frac{1}{2}$ $P(B) = \frac{1}{2}$ & $P(A \cap B) = \frac{1}{6}$ ½

Now $P(A \cap B) = \frac{1}{6} = P(A) \cdot P(B)$ for independent ½

$\Rightarrow P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \neq P(A \cap B)$

So that A & B are not independent. ½

11) Minimum labour cost, $Z = 300x + 400y$

Subject to constraints: 1

$6x + 10y \leq 60$, $4x + 4y \leq 32$, $x, y \geq 0$ 1

Where x be the no. of stitch shirts & y be trousers.

12) $$\int_{\pi/4}^{\pi/2} \sqrt{1 - \sin 2x}\, dx = \int_{\pi/4}^{\pi/2} \sqrt{\sin^2 x + \cos^2 x - 2\sin x \cos x}\, dx$$ 1

$$= \int_{\pi/4}^{\pi/2} \sqrt{(\sin x - \cos x)^2}\, dx = \int_{\pi/4}^{\pi/2} \sin x - \cos x\, dx$$ ½

$$= -\left[\cos x + \sin x\right]_{\pi/4}^{\pi/2} = \sqrt{2} - 1$$ ½

13. L.H.S $\left(\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}\right)+\sin^{-1}\frac{16}{65}$ ½

$=\sin^{-1}\left[\frac{4}{5}\sqrt{1-\left(\frac{5}{13}\right)^2}+\frac{5}{13}\sqrt{1-\left(\frac{4}{5}\right)^2}\right]+\sin^{-1}\frac{16}{65}$ 1½

$=\sin^{-1}\left(\frac{4}{5}\sqrt{\frac{144}{169}}+\frac{5}{13}\sqrt{\frac{9}{25}}\right]+\sin^{-1}\frac{16}{65}$

$=\sin^{-1}\left(\frac{48}{65}+\frac{15}{65}\right)+\sin^{-1}\frac{16}{65}$ 1

$=\sin^{-1}\frac{63}{65}+\cos^{-1}\left[\sqrt{1-\left(\frac{16}{25}\right)^2}\right]$ 1

$=\sin^{-1}\frac{63}{65}+\cos^{-1}\frac{63}{65}=\pi/2$ $\left[\sin^{-1}x+\cos^{-1}x=\pi/2\right]$

OR

Given

$\tan^{-1}2x+\tan^{-1}3x=\pi/4$

$\tan^{-1}\left(\frac{2x+3x}{1-2x.3x}\right)=\frac{\pi}{4}$ $\left(\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$ 1

$\Rightarrow \frac{5x}{1-6x^2}=1$ 1

$\Rightarrow 6x^2+5x-1=0$ 1

$\Rightarrow (6x-1)(x+1)=0$

$\Rightarrow x=1/6\ ,\ -1\ ;\ x=-1$ (rejected) 1

Hence $x=1/6$ is required solution

14) We have Equation of lines

$\frac{1-x}{3}=\frac{7y-14}{2\lambda}=\frac{5z-10}{11}$ ..

$\Rightarrow \frac{x-1}{-3}=\frac{y-2}{\frac{2\lambda}{7}}=\frac{z-2}{11/5}$ (in Standard form) ½ —①

& $\frac{x-1}{\frac{-3\lambda}{7}}=\frac{y-5}{1}=\frac{z-6}{-5}$ —② ½

When two lines are perpendicular,

$a_1a_2+b_1b_2+c_1c_2=0$

$$\Rightarrow \frac{9\lambda}{7} + \frac{2\lambda}{7} - \frac{55}{5} = 0$$

$$\Rightarrow 11\lambda = 70 \Rightarrow \lambda = \frac{70}{11}$$

15) $\frac{dy}{dx} + y = \cos x - \sin x$

$\frac{dy}{dx} + Py = Q$ 1/2

$I.F = e^{\int P\,dx} = e^{\int dx} = e^{x}$

Solution is $y \times I.F = \int (Q \times I.F)\,dx$ 1/2

$$\Rightarrow y \cdot e^{x} = \int e^{x}(\cos x - \sin x)\,dx$$

$$\Rightarrow y \cdot e^{x} = \int e^{x}\cos x\,dx - \int e^{x}\sin x\,dx$$ 1½

$$\Rightarrow y \cdot e^{x} = \cos x \int e^{x}dx - \int\left\{\int e^{x}dx \times \frac{d}{dx}\cos x\right\}dx - \int e^{x}\sin x\,dx$$

$$\Rightarrow y \cdot e^{x} = e^{x}\cos x + \cancel{\int e^{x}\sin x\,dx} - \cancel{\int e^{x}\sin x\,dx} + C$$ 1½

$$\Rightarrow y = \cos x + c \cdot e^{-x} \quad \underline{\text{Ans.}}$$

16) We have $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\frac{y}{x} = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \operatorname{cosec}\frac{y}{x}$$ 1½

Let $y = vx$

$$\frac{dy}{dx} = v \cdot 1 + x\frac{dv}{dx}$$

$$\Rightarrow \cancel{v} + x\frac{dv}{dx} = \cancel{v} - \operatorname{cosec} v$$ 1½

$$\Rightarrow \int \frac{dv}{\operatorname{cosec} v} = -\int \frac{dx}{x}$$

$$\Rightarrow -\cos v = -\log x + c$$ 1/2

$$\Rightarrow \log x - \cos\left(\frac{y}{x}\right) = c \quad \underline{\text{Ans}}.$$

When $x = 1,\ y = 0 \Rightarrow 0 - 1 = c$ 1/2

$\therefore$ Particular Solution $\log x - \cos\left(\frac{y}{x}\right) = -1$ Ans.

⑤

$L.H.S = \Delta = \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$

Using $R_1 \to R_1 + R_2 + R_3$ 1

$\Delta = \begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$

$= (5x+4) \begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$ 1

Using $C_1 \to C_1 - C_3$ & $C_2 \to C_2 - C_3$ 1

$= (5x+4) \begin{vmatrix} 0 & 0 & 1 \\ 0 & 4-x & 2x \\ x-4 & x-4 & x+4 \end{vmatrix}$

$= (5x+4)(4-x)^2 \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 2x \\ -1 & -1 & x+4 \end{vmatrix}$ 1

By Expanding along R_1

$= (5x+4)(4-x)^2 \times 1 = (5x+4)(4-x)^2 = R.H.S$

proved.

18) Slope of tangent $\frac{dy}{dx} = \frac{1 \times 3}{2\sqrt{3x-2}}$ is ||el to ½

line $4x - 2y + 5 = 0$

their slope $= -\frac{a}{b} = \frac{-4}{-2} = 2$ ½

$\Rightarrow m_1 = m_2$

$\Rightarrow \frac{3}{2\sqrt{3x-2}} = 2$ 1

$\Rightarrow 3 = 4\sqrt{3x-2}$

$\Rightarrow 9 = 16(3x-2) \Rightarrow 48x = 41$

$\Rightarrow x = 41/48$

from ① $y = \sqrt{3x-2}$

$y = \sqrt{\cancel{3}\cdot\frac{41}{\cancel{48}_{16}} - 2} = \sqrt{\frac{9}{16}} = \frac{3}{4}$ ½

Equation of tangent, ½

$y - \frac{3}{4} = 2\left(x - \frac{41}{48}\right)$

$\Rightarrow \frac{4y-3}{\cancel{4}} = \frac{2(48x-41)}{\cancel{48}\,12}$ 1

$\Rightarrow 48y - 36 = 96x - 82$

Req. equⁿ is $96x - 48y - 46 = 0$

$\Rightarrow \boxed{48x - 24y - 23 = 0}$ Ans.

OR

We have $f(x) = x^3 + \frac{1}{x^3}$ 1

$f'(x) = 3x^2 - \frac{3}{x^4} = 0 \quad 3x^2 = \frac{3}{x^4}$ 1

$\Rightarrow x^6 - 1 = 0 \Rightarrow (x^2-1)(x^4+x^2+1) = 0$

$\Rightarrow x = \pm 1$ or $x^4+x^2+1 \neq 0$.

Draw Sign Scheem, 1

$$\begin{array}{ccccccc} & +ve & & -ve & & +ve & \\ -\infty & & -1 & & 1 & & \infty \end{array}$$

Increasing intervals $]-\infty, -1] \cup [1, \infty[$ 1

Decreasing intervals $[-1, 1]$

19) Here $p = P$(throwing a 'Six' in a Single throw) $= \frac{1}{6}$

$q = 1 - p = 5/6$

Req. prob. $= P$(throwing 'six' twice in 5 throws) $\times$

P(throwing 'six' in 6th throw)

$= {}^5C_2\, p^2 q^3 \times \frac{1}{6}$

$$= \frac{5\times 4}{1\times 2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3\times\frac{1}{6} = \frac{625}{23328}.$$

20) We have $y = x^{\sin x} + (\sin x)^{\cos x}$

$\Rightarrow y = u + v$ (let) ½

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

Now $u = x^{\sin x}$

Taking log on both sides, 1½

$\log u = \sin x \log x$

$$\frac{1}{u}\frac{du}{dx} = \frac{\sin x}{x} + \log x \cos x$$

$$\Rightarrow \frac{du}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \log x.\cos x\right)$$

Again $v = (\sin x)^{\cos x}$

$\log v = \cos x \log \sin x$ 1½

$$\frac{1}{v}\frac{dv}{dx} = \cos x \cdot \frac{1}{\sin x}\cos x + \log \sin x(-\sin x)$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x}\left[\cos x \cot x - \sin x \log \sin x\right]$$ ½

$$\therefore \text{Req. soln } \frac{dy}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \cos x \log x\right) + (\sin x)^{\cos x}\left[\cos x \cot x - \sin x \log \sin x\right]$$

21) $\int_2^4 2^x dx$ by limit sum.

We have $\int_a^b f(x)dx = \lim_{h\to 0} h \times \sum_{r=1}^{n} f(a+rh)$ 1

$nh = b - a = 2$

$f(x) = 2^x$ 1½

$f(a+rh) = f(2+rh) = 2^2 \cdot 2^{rh}.$

$$\therefore \int_2^4 2^x dx = \lim_{h\to 0} h\times\sum 4\cdot 2^{rh}$$

$$= \lim_{h\to 0} 4h\left[2^h + 2^{2h} + \cdots + 2^{nh}\right]$$ 1½

$$= \lim_{h\to 0} 4h\cdot\frac{(2^h)^n - 1}{2^h - 1} = \frac{4\cdot(2^2-1)}{\log 2} = \frac{12}{\log 2}$$ Ans