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Practice Text - 3

26.12.17

Marking Scheme (Mathematics XII 2017-18)

Sr. No.	Answer	Mark(s)
Section A		
1.	Yes J is symmetric	[1]
2.	-15	[1]
3.	Answer $[i j k] = \hat{i} \cdot (\hat{j} \times \hat{k}) = \hat{i} \cdot \hat{i} = 1$	[1]
4.	$(1*2)*3 = 2^2*3 = 2^{12}$, $1*(2*3) = 1*2^6 = 2^{64} \therefore (1*2)*3 \neq 1*(2*3)$. Hence, * is not associative.	[1]
Section B		
5.	$\Rightarrow 4 \sin^{-1} x + \frac{\pi}{2} - \sin^{-1} x = \pi$ $\Rightarrow 3 \sin^{-1} x = \frac{\pi}{2} \Rightarrow \sin^{-1} x = \frac{\pi}{6} \Rightarrow x = \sin \frac{\pi}{6} = \frac{1}{2}$	[1] [1]
6.	$\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1} = \frac{1}{9-10} \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$ $P = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 13 & 8 \\ 1 & 1 \end{bmatrix}$	[1 + ½] [½]
12.	$\tan^{-1} \left(\frac{\cos x}{1 + \sin x} \right) = \tan^{-1} \left(\frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{(\cos \frac{x}{2} + \sin \frac{x}{2})^2} \right)$ $= \tan^{-1} \left[\frac{\cos \frac{x}{2} - \sin \frac{x}{2}}{\cos \frac{x}{2} + \sin \frac{x}{2}} \right] = \tan^{-1} \left[\frac{1 - \tan \frac{x}{2}}{1 + \tan \frac{x}{2}} \right]$ $= \tan^{-1} \tan \left(\frac{\pi}{4} - \frac{x}{2} \right) = \left(\frac{\pi}{4} - \frac{x}{2} \right)$	[½] [1]
	3	[½]
7.	Let $y = \frac{1}{x^2}$. Then $\frac{dy}{dx} = \frac{-2}{x^3}$. $dy = \left(\frac{dy}{dx} \right)_{x=2} \times \Delta x = \frac{-2}{2^3} \times 0.002 = -0.0005$. y decreases by 0.0005.	[1/2] [1] [1/2]
8.	$\int e^x \frac{\sqrt{1 + \sin 2x}}{1 + \cos 2x} dx = \int e^x \frac{\sqrt{(\sin x + \cos x)^2}}{2 \cos^2 x} dx$ $= \frac{1}{2} \int e^x \left(\frac{\sin x}{\cos^2 x} + \frac{\cos x}{\cos^2 x} \right) dx = \frac{1}{2} \int e^x (\sec x + \sec x \tan x) dx$ $= \frac{1}{2} e^x \sec x + c \left[\because \int e^x (f(x) + f'(x)) dx = e^x f(x) + c \right]$	[1/2] [1/2] [1]

10. 9	$ax^2 + by^2 = 1 \Rightarrow 2ax + 2byy_1 = 0 \Rightarrow ax + byy_1 = 0 \quad (1)$ $\Rightarrow a + b[yy_2 + y_1^2] = 0 \Rightarrow a = -b[yy_2 + y_1^2] \quad (2)$ Substituting this value, for a in the equation (1), we get, $-b[yy_2 + y_1^2]x + byy_1 = 0 \Rightarrow x[yy_2 + y_1^2] = yy_1$. Hence verified	[1/2] [1] [1/2]
11. 10	$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} - 2\hat{j} + \hat{k}, \vec{a} \cdot \vec{b} = 5, \vec{b} = \sqrt{6}$. The required Projection (vector) of \vec{a} on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{ \vec{b} ^2} \vec{b}$ $= \frac{5}{6}(\hat{i} - 2\hat{j} + \hat{k})$.	[1/2] [1] [1/2]
12. 11.	$P(B/A) = \frac{P(A \cap B)}{P(A)} \Rightarrow P(A \cap B) = 0.4 \times 0.6 = 0.24$ $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.24}{0.80} = \frac{3}{10}$	[1] [1]
Section C		
13.	$\text{Let } \Delta_1 = \begin{vmatrix} a^3 - 1 & 0 & a - a^4 \\ 0 & a - a^4 & a^3 - 1 \\ a - a^4 & a^3 - 1 & 0 \end{vmatrix} = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$ Where C_{ij} = the cofactor of a_{ij} and a_{ij} = the (i, j)th element of determinant Δ . $\text{We know that } \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix} = \Delta^2$ $\therefore \Delta_1 = \Delta^2 = (-4)^2 = 16$	[2] [1] [1]
14.	Since, f is differentiable at 1, f is continuous at 1. Hence, $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (2x + 1) = 3$ $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (ax^2 + b) = a + b$ $f(1) = 3$ As f is continuous at 1, we have $a + b = 3 \dots (1)$ $Lf'(1) = \lim_{h \rightarrow 0^+} \frac{f(1-h) - f(1)}{-h} = \lim_{h \rightarrow 0^+} \frac{a(1-h)^2 + b - 3}{-h}$ $= \lim_{h \rightarrow 0^+} \frac{a + ah^2 - 2ah + b - 3}{-h} = \lim_{h \rightarrow 0^+} (-ah + 2a) \text{ (using (1))}$ $= 2a$ $Rf'(1) = \lim_{h \rightarrow 0^+} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0^+} \frac{2(1+h) + 1 - 3}{h} = 2$ As f is differentiable at 1, we have $2a = 2$, i. e., $a = 1$ and $b = 2$.	[1] [1/2] [1/2] [1/2] [1/2] [1/2] [1/2]
OR		

	$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{x + \sin x}{\sin(a+1)x}$ $= \lim_{x \rightarrow 0^-} \frac{1 + \frac{\sin x}{x}}{\frac{\sin(a+1)x}{(a+1)x}} = \frac{2}{a+1}$ $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} 2 \frac{e^{\sin bx} - 1}{bx}$ $= \lim_{x \rightarrow 0^+} 2 \frac{e^{\sin bx} - 1}{\sin bx} \times \frac{\sin bx}{bx} = 2$ <p>$f(0) = 2.$</p> <p>For the function to be continuous at 0, we must have $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x) = f(0)$</p> <p>i.e., we must have $\frac{2}{a+1} = 2 \Rightarrow a = 0$; b may be any real number other than 0.</p>	<p>[1/2]</p> <p>[1/2]</p> <p>[1/2]</p> <p>[1/2]</p> <p>[1/2]</p> <p>[1/2]</p> <p>[1/2]</p>
15.	$y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 = 2 \log\left(\frac{x+1}{\sqrt{x}}\right) = 2\left[\log(x+1) - \frac{1}{2} \log x\right]$ $y_1 = 2\left[\frac{1}{x+1} - \frac{1}{2} \times \frac{1}{x}\right] = \frac{x-1}{x(x+1)} \quad (1)$ $y_2 = \frac{x(x+1) - (x-1)(2x+1)}{x^2(x+1)^2} = \frac{-x^2 + 2x + 1}{x^2(x+1)^2}$ $\Rightarrow x(x+1)^2 y_2 = \frac{-x^2 + 2x + 1}{x} = \frac{2x - (x+1)(x-1)}{x} = 2 - (x+1)^2 y_1 \quad (\text{using (1)})$ $\Rightarrow x(x+1)^2 y_2 + (x+1)^2 y_1 = 2. \text{ Hence, proved.}$	<p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p>
16.	<p>When $y = 0$, we have $(x-1)(x^2+x+1)(x-2) = 0$, i.e., $x = 1$ or 2.</p> $\frac{dy}{dx} = x^3 - 1 + (x-2)3x^2 = 4x^3 - 6x^2 - 1$ $\left(\frac{dy}{dx}\right)_{(1,0)} = -3$ $\left(\frac{dy}{dx}\right)_{(2,0)} = 7.$ <p>The required equations of the tangents are $y - 0 = -3(x-1)$ or, $y = -3x + 3$ and $y - 0 = 7(x-2)$ or, $y = 7x - 14$.</p> <p style="text-align: center;">OR</p> <p>Domain $f = (-1, \infty)$ $f'(x) = \frac{-3}{1+x} + \frac{4}{(2+x)} + \frac{4}{(2+x)^2} = \frac{x(x+4)}{(1+x)(2+x)^2}$.</p> $f'(x) = 0 \Rightarrow x = 0 \text{ [} x \neq -4 \text{ as } -4 \notin (-1, \infty)\text{].}$ <p>In $(-1, 0)$, $f'(x) = \frac{(-ve)(+ve)}{(+ve)(+ve)} = -ve$. Therefore, f is strictly decreasing in $(-1, 0)$.</p> <p>In $(0, \infty)$, $f'(x) = +ve$. Therefore, f is strictly increasing in $[0, \infty)$.</p>	<p>[1/2]</p> <p>[1/2]</p> <p>[1/2]</p> <p>[1/2]</p> <p>[2]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p>

17.

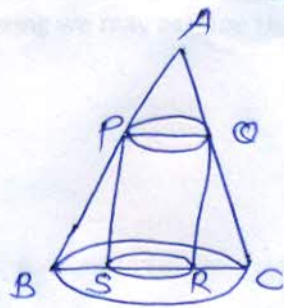
Let $AL = x$ $MC = x$ $AM = R$ (Given) $LM = R - x$ $\triangle ALO \sim \triangle AMC$

$$LO = \frac{x^2}{R}$$

$$V = \pi \frac{x^2 - x^2}{R^2} (R - x)$$

$$\frac{dV}{dx} = \frac{\pi x^2}{R^2} (2R - 3x) = 0 \quad x = \frac{2R}{3}$$

$$\frac{d^2V}{dx^2} = \frac{\pi x^2}{R^2} (-2R) < 0 \text{ at } x = \frac{2R}{3}$$

Height of cylinder = $R - x = R - \frac{2R}{3} = \frac{R}{3}$ 

[1]

[1]

[1]

[1]

18.

$$\int \frac{\sec x}{1 + \cos e^x} dx = \int \frac{\sin x}{\cos x(1 + \sin x)} dx = \int \frac{\sin x \cos x}{(1 + \sin x)^2(1 - \sin x)} dx$$

$$= \int \frac{t}{(1+t)^2(1-t)} dt \quad [\sin x = t \Rightarrow \cos x dx = dt]$$

$$\frac{t}{(1+t)^2(1-t)} = \frac{A}{1+t} + \frac{B}{(1+t)^2} + \frac{C}{1-t} \Rightarrow t = A(1+t)(1-t) + B(1-t) + C(1+t)^2$$

(an identity)

Put $t = -1$, $-1 = 2B$, i.e., $B = -\frac{1}{2}$. Put $t = 1$, $1 = 4C$, i.e., $C = \frac{1}{4}$. Put $t = 0$, $0 = A + B + C$, which gives $A = \frac{1}{4}$.

[1 + 1/2]

$$\text{Therefore the required integral} = \frac{1}{4} \int \frac{1}{1+t} dt + \frac{-1}{2} \int \frac{1}{(1+t)^2} dt + \frac{1}{4} \int \frac{1}{1-t} dt$$

$$= \frac{1}{4} \log|1+t| + \frac{-1}{2} \times \frac{-1}{1+t} - \frac{1}{4} \log|1-t| + c$$

$$= \frac{1}{4} \log|1 + \sin x| + \frac{1}{2} \times \frac{1}{1 + \sin x} - \frac{1}{4} \log|1 - \sin x| + c$$

[1 + 1/2]

19.

The given differential equation is $ye^y dx = (y^3 + 2xe^y) dy$, $y(0) = 1$ or, $\frac{ye^y}{(y^3 + 2xe^y)} = \frac{dy}{dx}$ or, $\frac{dx}{dy} + (-\frac{2}{y})x = \frac{y^2}{e^y}$, which is linear in x .

$$\text{I. F.} = e^{\int \frac{-2}{y} dy} = e^{-2 \log y} = \frac{1}{y^2}$$

[1]

Multiplying both sides by the I. F. and integrating, we get, $x \frac{1}{y^2} = \int e^{-y} dy$

[1/2]

$$\Rightarrow x \frac{1}{y^2} = -e^{-y} + c \Rightarrow x = -y^2 e^{-y} + cy^2 \text{ (the general solution).}$$

[1]

When $x = 0$, $y = 1$. $0 = -e^{-1} + c \Rightarrow c = \frac{1}{e}$. Hence, the required particular solution is

$$x = -y^2 e^{-y} + \frac{y^2}{e}$$

[1/2]

OR

The given differential equation is $(x-y)dy = (x+2y)dx$ or, $\frac{dy}{dx} = \frac{x+2y}{x-y} = \frac{1+2\frac{y}{x}}{1-\frac{y}{x}} = f\left(\frac{y}{x}\right)$, [1]

hence, homogeneous. [1]

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$. The equation becomes $v + x \frac{dv}{dx} = \frac{1+2v}{1-v}$ or, $\frac{1-v}{v^2+v+1} dv = \frac{dx}{x}$

or, $\frac{-1}{2} \times \frac{2v+1-3}{v^2+v+1} dv = \frac{dx}{x}$ or, $\left[\frac{2v+1}{v^2+v+1} + \frac{-3}{v^2+2v \times \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \frac{3}{4}} \right] dv = \frac{-2dx}{x}$

Integrating, we get $\int \frac{2v+1}{v^2+v+1} dv + \int \frac{-3}{\left(v+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dv = \int \frac{-2dx}{x}$

or, $\log(v^2+v+1) - \frac{3 \times 2}{\sqrt{3}} \tan^{-1} \frac{v+\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -2 \log|x| + c$

or, $\log(y^2+xy+x^2) - 2\sqrt{3} \tan^{-1} \frac{2y+x}{\sqrt{3}x} = c$ (the general solution). [2]

20. $\vec{a} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{a} \times \vec{0}$ [1]

$\Rightarrow \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = \vec{0} \Rightarrow \vec{a} \times \vec{b} = \vec{c} \times \vec{a}$ [1]

$\vec{b} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{b} \times \vec{0}$ [1/2]

$\Rightarrow \vec{b} \times \vec{c} = \vec{a} \times \vec{b} \therefore \vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$ [1/2]

$[\vec{a} \ \vec{b} \ \vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{a} \cdot (\vec{a} \times \vec{b})$ [1/2]

$= 0$ [As the scalar triple product of three vectors is zero if any two of them are equal.] [1/2]

21. Equation of plane pt $(1, 4, 6)$ is [1/2]

$a(x-1) + b(y-4) + c(z-6) = 0$ — (1) [1/2]

is \perp to line $\frac{x-1}{4} = \frac{y-2}{5} = \frac{z-3}{6}$ [1/2]

but Normal to plane \parallel to line

$\frac{a}{4} = \frac{b}{5} = \frac{c}{6} = k$

$a = 4k \quad b = 5k \quad c = 6k$

Put (1)

$4k(x-1) + 5k(y-4) + 6k(z-6) = 0$ [1]

$4(x-1) + 5(y-4) + 6(z-6) = 0 \quad k \neq 0$ [1/2]

$4x - 4 + 5y - 20 + 6z - 36 = 0$ [1/2]

$4x + 5y + 6z - 60 = 0$

22. Let E_1 = Bag I is chosen, E_2 = Bag II is chosen, E_3 = Bag III is chosen, A = The two balls drawn from the chosen bag are white and red.

$P(E_1) = \frac{1}{3} = P(E_2) = P(E_3)$,

$P(A|E_1) = \frac{1}{6} \times \frac{3}{6} \times 2, P(A|E_2) = \frac{2}{4} \times \frac{1}{4} \times 2, P(A|E_3) = \frac{4}{9} \times \frac{2}{9} \times 2$.

By Bayes's Theorem, the required probability =

$$P(E_3|A) = \frac{P(E_3) \times P(A|E_3)}{\sum_{i=1}^3 P(E_i) \times P(A|E_i)} = \frac{\frac{1}{3} \times \frac{4}{9} \times \frac{2}{9} \times 2}{\frac{1}{3} \times \frac{1}{6} \times \frac{3}{6} \times 2 + \frac{1}{3} \times \frac{2}{4} \times \frac{1}{4} \times 2 + \frac{1}{3} \times \frac{4}{9} \times \frac{2}{9} \times 2} = \frac{64}{199}$$

23. Let X denotes the random variable. Then $X = 0, 1, 2$.

$P(X=0) = \frac{{}^{16}C_2}{{}^{20}C_2} = \frac{60}{95}, P(X=1) = \frac{{}^4C_1 \times {}^{16}C_1}{{}^{20}C_2} = \frac{32}{95}, P(X=2) = \frac{{}^4C_2}{{}^{20}C_2} = \frac{3}{95}$.

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
0	60/95	0	0
1	32/95	32/95	32/95
2	3/95	6/95	12/95
total		38/95	44/95

Mean = $\sum_{i=1}^3 x_i p_i = \frac{38}{95} = \frac{2}{5}$

Variance = $\sum_{i=1}^3 x_i^2 p_i - (\sum_{i=1}^3 x_i p_i)^2 = \frac{44}{95} - \frac{4}{25} = \frac{144}{475}$.

Section D

24. $f \circ g : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f \circ g(x) = f(g(x)) = f(x^3 + 5) = 2(x^3 + 5) - 3 = 2x^3 + 7$

Let $x_1, x_2 \in \mathbb{R}(D_{f \circ g})$ such that

$f \circ g(x_1) = f \circ g(x_2) \Rightarrow 2x_1^3 + 7 = 2x_2^3 + 7 \Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2$. Hence, $f \circ g$ is one-one.

Let $y \in \mathbb{R}(\text{Codomain}_{f \circ g})$. Then for any x $f \circ g(x) = y$ if $2x^3 + 7 = y$, i.e., if, $2x^3 = y - 7$, i.e., $x = \sqrt[3]{\frac{y-7}{2}}$, which $\in \mathbb{R}(D_{f \circ g})$. Hence, for every $y \in \mathbb{R}(\text{Codomain}_{f \circ g}), \exists \sqrt[3]{\frac{y-7}{2}} \in \mathbb{R}(D_{f \circ g})$

such that $f \circ g(\sqrt[3]{\frac{y-7}{2}}) = y$. Hence, $f \circ g$ is onto.

Since, $f \circ g$ is both one-one and onto, it is invertible.

$$(f \circ g)^{-1} : \mathbb{R} \rightarrow \mathbb{R} \text{ defined by } (f \circ g)^{-1}(x) = \sqrt[3]{\frac{x-7}{2}}$$

[1]

$$(f \circ g)^{-1}(9) = \sqrt[3]{\frac{9-7}{2}} = 1.$$

[1/2]

OR

Let $a, b \in \mathbb{R}$ such that $a = 0, b \neq 0$.

$$\text{Then } a * b = |a| + b = |0| + b = b, b * a = b, \therefore a * b = b * a$$

[1]

Let $a, b \in \mathbb{R}$ such that $a \neq 0, b = 0$.

[1]

$$\text{Then } a * b = a, b * a = |b| + a = |0| + a = a, \therefore a * b = b * a$$

[1]

Let $a, b \in \mathbb{R}$ such that $a = 0, b = 0$. Then $a * b = a = 0, b * a = b = 0, \therefore a * b = b * a$.

Now we need to check whether $*$ is commutative. One more case is needed to be

examined. Let $a, b \in \mathbb{R}$ such that $a \neq 0, b \neq 0$. Then $a * b = |a| + b, b * a = |b| + a$ and $a * b$ may not be equal to $b * a$, e.g., $(-1) * 2 = 3, 2 * (-1) = 1$, hence, $(-1) * 2 \neq 2 * (-1)$. Thus $*$ is not commutative.

[1]

The element $e \in \mathbb{R}$ will be the identity element for $*$ if $a * e = e * a = a$ for all $a \in \mathbb{R}$.

$a * e = a$ provided $e = 0$ and $e * a = a$ provided $e = 0$ (As $0 * 0 = 0$ and $0 * a = |0| + a = a$ for $a \neq 0$). Hence, 0 is the identity element for $*$.

[2]

23.

$$|A| = 3(3-6) + (-2)(-12-14) + 1(12+7) = 62 \neq 0.$$

[1]

Hence, A^{-1} exists. Let c_{ij} represent the cofactor of $(i, j)^{\text{th}}$ element of A. Then,

$$c_{11} = -3, c_{12} = 26, c_{13} = 19, c_{21} = 9, c_{22} = -16, c_{23} = 5, c_{31} = 5, c_{32} = -2, c_{33} = -11.$$

$$\text{adj}A = \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$$

$$A^{-1} = \frac{1}{62} \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$$

[2]

The given system of equations is equivalent to the matrix equation

$$A'X = B, \text{ where } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}.$$

$$\Rightarrow X = (A')^{-1}B = (A^{-1})'B$$

[1]

$$= \frac{1}{62} \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix} \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix} = \frac{1}{62} \begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ Hence, } x = 1, y = 1, z = 1$$

[2]

OR

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A (R_1 \leftrightarrow R_2)$$

[1]

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} A (R_2 \rightarrow R_2 - 2R_1)$$

[1]

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ -2 & 4 & 1 \end{bmatrix} A (R_3 \rightarrow R_3 - 2R_2)$$

[1]

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -3 & -1 \\ -1 & 2 & 1 \\ -2 & 4 & 1 \end{bmatrix} A (R_2 \rightarrow R_2 + R_3, R_1 \rightarrow R_1 - R_3)$$

[1]

$$\Rightarrow A^{-1} = \begin{bmatrix} 2 & -3 & -1 \\ -1 & 2 & 1 \\ -2 & 4 & 1 \end{bmatrix}$$

[1]

$$X = [1 \ 0 \ 1]A^{-1} = [1 \ 0 \ 1] \begin{bmatrix} 2 & -3 & -1 \\ -1 & 2 & 1 \\ -2 & 4 & 1 \end{bmatrix} = [0 \ 1 \ 0]$$

[1]

26.

$$y = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$$

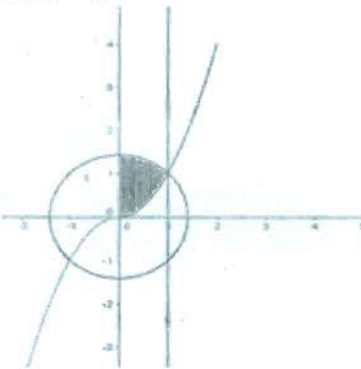
[1]

Solving $y = x^2, x^2 + y^2 = 2$ simultaneously, $y + y^2 - 2 = 0 \Rightarrow (y+2)(y-1) = 0 \Rightarrow y = 1$

($y = x^2$ lies in quadrant I).

$$\Rightarrow x = 1$$

[1/2]



[1]

The required area = the shaded area = $\int_0^1 (\sqrt{2-x^2} - x^2) dx$

[2]

$$= \frac{1}{2} [x\sqrt{2-x^2} + 2\sin^{-1} \frac{x}{\sqrt{2}}]_0^1 - \frac{1}{3} [x^3]_0^1 = (\frac{1}{6} + \frac{\pi}{4}) \text{ sq units.}$$

[1+1/2]

