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DAV PUBLIC SCHOOL, HEHAL, RANCHI

Sub. Maths

Practice Set-2 Class X

Marking Scheme

1. $\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta ADE} = \left(\frac{AB}{AD}\right)^2 = \left(\frac{AD+BD}{AD}\right)^2 = \left(\frac{1+2}{1}\right)^2 = \frac{9}{1}$ $(\because \Delta ABC \sim \Delta ADE)$

   $\therefore$ Area of Δ ABC : Area of ΔADE = 9: 1

2. PC = PA – CA = PB – CQ = (10-2) cm = 8 cm $(\because$ PA = PB and CA = CQ)
3. Given AB=9 => $(a + 3)^2 + (-5 + 14)^2 = 9$

   $\Rightarrow$ $(a+3)^2 = 0$

   $\Rightarrow$ a= -3

4.Tan θ = cot (30° + θ) =tan [90° - (30° +θ)] = tan (60° -θ)

$\therefore \theta = 60° - \theta \Longrightarrow 20 = 60° \Longrightarrow \theta = 30°$.

5. Distance = $\sqrt{\left(\frac{2}{5}+\frac{8}{5}\right)^2 + (2-2)^2} = \sqrt{\left(\frac{10}{5}\right)^2 + 0} = \sqrt{4+0} = 2$

Hence, the distance between $\left(-\frac{8}{5}, 2\right)$ and $\left(\frac{2}{5}, 2\right)$ is 2 units.

6Distance between parallel tangents = (4+ 4) cm = 8 cm.

7 $\frac{\cos 70°}{\sin 20°}$ +cos 57° . cosec 33° -2 cos 60° = $\frac{\cos 70°}{\sin(90°-70°)}$ + cos 57° . cosec (90°-57°) - 2× $\frac{1}{2}$

= $\frac{\cos 70°}{\cos 70°}$ +cos 57° . sec 57° -1 = 1+ $\frac{\cos 57°}{\cos 57°}$ – 1= 1+ 1 -1 = 1.

8. Given , ΔABC *is* an isosceles triangle with AB =AC.

   $\Longrightarrow \angle C = \angle B$ (angle opp. Equal sides of a Δ are equal)

   In ΔABD and ΔECF, $\angle ABD = \angle ECF$ $(\because \angle B = \angle c$,prove above)

   and $\angle ADB = \angle EFC$(each =90°) $\therefore$ ΔABD~Δ ECF (by AA similarity criterion)

   $\therefore \frac{AB}{EC} = \frac{AD}{EF} \Longrightarrow$ AB × EF = AD × EC , as required.

9. In ΔABC, DE ∥ BC by cor. To B.P.T, We have

   $\frac{DB}{AB} = \frac{EC}{AC} \Longrightarrow \frac{x-3}{2x} = \frac{x-2}{2x+3} \Longrightarrow 2x(x-2) = (x-3)(2x+3) \Rightarrow 2x^2 - 4x = 2x^2 + 3x - 6x - 9$

   $\Longrightarrow$ -4x = -3x-9 $\Longrightarrow$ -x = -9 $\Longrightarrow$ x = 9 Hence , the value of x is 9 .

10. Since G (4,3) is the centroid of ΔABC , we have $\frac{1+4+a}{3}$ = 4 and $\frac{3+b+1}{3}$ =3

    $\Longrightarrow$ 5 +a = 12 and 4+ b =9 $\Longrightarrow$ a= 7 and b = 5. Hence , the value of a and b are a =7, b = 5.

    The coordinates of the point B and C are (4, 5) and (7, 1) respectively.

    Length of side BC = $\sqrt{(7-4)^2 + (1-5)^2}$ units = $\sqrt{9+16}\ units = 5\ \boldsymbol{units}.$

11. In ΔABC , DE ∥ AC.

    $\therefore \frac{BE}{EC} = \frac{BD}{DA}$ (by B.P.T) But $\frac{BE}{EC} = \frac{BC}{CP}\ (Given) \Longrightarrow \frac{BD}{DA} = \frac{BC}{CP} \Longrightarrow$ DC∥ AP (Converse of B.P.T.)

12. AR = AQ , DR = DS , BP = BQ (lengths of tangents)

    But DS = 5cm $\Longrightarrow$ DR = 5 cm. AR =AD –DR =23 cm – 5 cm =18 cm $\Longrightarrow$ AQ = 18 cm.

    BQ = AB – AQ = 29 cm – 18 cm =11 cm. As AB is tangent to the circle at Q , OQ ⊥ AB

    $\Longrightarrow$ ∠OQB = 90°. Also ∠B = 90°(Given) $\Longrightarrow$ OQBP is a rectangle . But BP = BQ $\Longrightarrow$ OQBP is a square.

    $\therefore$ Radius = r = OQ = BQ = 11 cm.

13. LHS = $\sqrt{\frac{secA-1}{secA+1}}+\sqrt{\frac{secA+1}{secA-1}}=\frac{(secA-1)+(secA+1)}{\sqrt{secA+1}.\sqrt{secA-1}}=\frac{2\sec A}{\sqrt{sec^2A-1}}=\frac{2secA}{\sqrt{tan^2A}}=\frac{2secA}{tanA}=2\times\frac{1}{cosA}\times\frac{cosA}{sinA}$

$=\frac{2}{sinA}$ = 2 cosec A = RHS

14. LHS = (1+cot A – cosec A) (1+tan A + sec A) = $\left(1+\frac{cosA}{sinA}-\frac{1}{sinA}\right)\left(1+\frac{sinA}{cosA}+\frac{1}{cosA}\right)$

$=\left[\frac{sinA+cosA-1}{sinA}\right]\left[\frac{cosA+sinA+1}{cosA}\right]=\frac{(sinA+cosA)^2-1}{sinA\ cosA}=\frac{sin^2A+cos^2A+2\sin A\cos A-1}{sinA\ cosA}$

$=\frac{1+2\sin A\cos A-1}{sinA\ cos\ A}=\frac{2\sin A\cos A}{sinA\cos A}$ =2 = RHS

15. In ΔACB and ΔCDB, ∠ACB = $\angle CDB$ (both 90°)$\angle B=\angle B$ (Common)

∴ Δ ACB ~ ΔCDB $\Longrightarrow\frac{AC}{CD}=\frac{AB}{CB}\Longrightarrow\frac{b}{p}=\frac{c}{a}\Longrightarrow\frac{1}{p}=\frac{c}{ab}$

Squaring both the sides, we get $\frac{1}{p^2}=\frac{c^2}{a^2b^2}$ Now, ΔABC is a right angled triangle.

∴ By Pythagoras theorem, $AB^2=BC^2+AC^2\Longrightarrow c^2=a^2+b^2$

Putting value of c^2 in (i), we get $\frac{1}{p^2}=\frac{a^2+b^2}{a^2b^2}\Longrightarrow\frac{1}{p^2}=\frac{1}{b^2}+\frac{1}{a^2}$

16. Given a circle with centre O and PQ is tangent to the circle at the point Q from an external point P. Chord QA is parallel to PO and AOB is a diameter. We need to prove that PB is tangent to the circle at the point B. Join OQ and mark the angles as shown in the adjoining figure.

As QA ∥ PO, ∠1 = ∠2 (alt.$\angle s$) and ∠4 = ∠3 (corres. ∠s)

But ∠2 = ∠3 (∵ in ΔOAQ, OA = OQ being radii) ∴ ∠1 = ∠ 4.

In ΔOPB and Δ OPQ, OB = OQ (*radii of same circle*) ∠1 = ∠4(Proved above)

OP = OP (Common) ∴ ΔOPB ≅ Δ OPQ (SAS congruence rule)

∴ ∠OBP = ∠OQP (c.p.c.t) ⟹ ∠OBP = 90° (tangent is⊥to radius, OQ ⊥ PQ)

⟹ OB ⊥ PB ie radius is perpendicular to PB at point B. Therefore , PB is tangent to the circle at the point B.

17. LHS = $\frac{1}{cosecA-\cot A}-\frac{1}{\sin A}=\frac{1}{cosecA-\cot A}\times\frac{(cosecA+\cot A)}{(cosec\ A+\cot A)}-\frac{1}{\sin\ A}$

$=\frac{cosecA+\cot A}{cosec^2A-cot^2\ A}-\frac{1}{sinA}$ =cosec A + cot A – Cosec A = cot A .

RHS = $\frac{1}{sin\ A}-\frac{1}{cosecA+\cot A}=\frac{1}{\sin A}-\frac{1}{cosec\ A+\cot A}\times\frac{(cosec\ A-\cot A)}{(cosecA-\cot A)}$

$=\frac{1}{sinA}-\frac{cosec\ A-\cot A}{cosec^2\ A-cot^2A}$ = cosec A – (cosec A – cot A) = cot A ∴ LHS = RHS

18. LHS = $\frac{\tan A+\sec A-1}{\tan A-\sec A+1}=\frac{\tan A+secA-(sec^2A-tan^2A)}{tanA-\sec A+1}$

$=\frac{\tan A+\sec A-(\sec A+tanA)(secA-\tan A)}{tanA-secA+1}=\frac{(tanA+secA)(1+secA-tanA)}{tanA-secA+1}$ = tan A +sec A

$=\frac{sinA}{cosA}+\frac{1}{cosA}=\frac{sinA+1}{cosA}=\frac{1+sinA}{cosA}=RHS$

19. Join AC. Now, area $(\Delta ABC)=\frac{1}{2}|-5(-6+1)+(-4)(-1+3)+2(-3+6)|$

$=\frac{1}{2}|25-8+6|=\frac{23}{2}\,sq.units$ .... (i) area(ΔADC) = $\frac{1}{2}|-5(2+1)+1(-1+3)+2(-3-2)|$

$=\frac{1}{2}|-15+2-10|=\frac{23}{2}\ sq.units$ ....(ii) Area (quad. ABCD) = Area $(\Delta ABC)+Area\ (\Delta ADC)$

$=\left(\frac{23}{2}+\frac{23}{2}\right)sq.units$ (from (i) and (ii) = 23 sq. units

20. LHS $\frac{tanA}{1+secA}-\frac{tanA}{1-secA}=\tan A\left(\frac{1}{1+secA}-\frac{1}{1-secA}\right)$ = tan A $\left(\frac{(1-secA)-(1+secA)}{1-sec^2A}\right)$

= tan A $\left(\frac{-2secA}{-tan^2A}\right)$ $(\because\ sec^2A-1=tan^2A)$

$=2\,\frac{secA}{tanA}=2.\frac{1}{cosA}.\frac{cosA}{sinA}=\frac{2}{sinA}$= 2cosecA = RHS

In ΔECD, $\angle ECD = 90°$ $\therefore DE^2 = CD^2 + EC^2$ ....(ii)
In ΔAEC, $\angle ACE = 90°$ $\therefore AE^2 = AC^2 + EC^2$ ....(iii)
In ΔBCD, $\angle BCD = 90°$ $\therefore BD^2 = BC^2 + CD^2$ ....(iv)
Adding (iii) and (iv), we get $AE^2 + BD^2 = (AC^2+BC^2) + (CD^2+EC^2) = AB^2 + DE^2$ [using (i) and (ii)

22. Since tangents drawn from an external point to a circle are equal.
$\therefore$ BF = BD = 4 cm (given) and CE = CD = 3 cm (given) Let AF = AE = x cm
Now, $area$ (ΔABC) = area $(\Delta AOB) + area\ b(\Delta BOC) + area\ (\Delta AOC)$
$\Rightarrow 21 = \frac{1}{2}\ AB \times OF + \frac{1}{2} BC \times OD + \frac{1}{2} AC \times OE \Rightarrow 21 = \frac{1}{2}(AF + BF) \times 2 + \frac{1}{2}(BD + CD)$
$\times 2 + \frac{1}{2}(AE + CE) \times 2$ ($\because$ OD = OE = OF = 2cm) $\Rightarrow 21 = (x+4) + (4+3) + (x+3)$
$\Rightarrow 21 = 2x + 14 \Rightarrow 2x = 7 \Rightarrow x = \frac{7}{2} \Rightarrow x = 3.5$
$\therefore$ AB = x+4 = (3.5+ 4)cm = 7.5 cm and AC = x+3 = (3.5 +3) = 6.5 cm .

23. Given. ABC is a right triangle right angled at A so that BC is its hypotenuse.
To Prove . $BC^2 = AB^2\ AC^2$. Construction. From A, draw AD$\perp$ BC.
Proof. In $\Delta DBA\ and\ \Delta ABC, \angle ABD = \angle ABC(same\ angle) and\ \angle ADB = \angle BAC(each = 90°$
$\therefore \Delta DBA \sim \Delta ABC$ $(AA\ similarity\ criterion)$ $\therefore \frac{AB}{BC} = \frac{BD}{AB} \Rightarrow AB^2 = BD \times BC$ ....(i)

In $\Delta DAC\ and\ \Delta ABC, \angle ACD = \angle ACB(same\ angle) and\ \angle ADC = \angle BAC(each\ 90°)$

$\therefore \Delta DAC \sim \Delta ABC(AA\ similarity\ criterion) \therefore \frac{AC}{BC} = \frac{DC}{AC} \Rightarrow AC^2 = DC \times BC$..............(ii)

On adding (i) and (ii) , we get $AB^2+AC^2 = BD \times BC + DC \times BC = (BD+DC) \times BC = BC \times BC$

$\Rightarrow AB^2 + AC^2 = BC^2$. Hence, $BC^2 = AB^2 + AC^2$.

24. Join OP, OQ and OC. Mark the angles as shown in the figure.
In $\Delta OAC\ and\ \Delta OAP$, OA = OA (Common) OC = OP (radii of same circle)
AC = AP (tangents drawn from A) $\therefore \Delta OAC \cong \Delta OAP(\ by\ SSS\ rule\ of\ congruency)$
$\therefore \angle 1 = \angle 2 \Rightarrow \angle PAC = 2\angle 1 \ldots\ldots\ldots (i) Similarly, \Delta OBC \cong \Delta OBQ,$
$\therefore \angle 3 = \angle 4 \Rightarrow \angle QBC = 2\angle 3 \ldots\ldots\ldots (ii)$
AS XY$\parallel$ X' Y' and AB is a transversal,
$\angle PAC + \angle QBC = 180°$,(sum of co- int.$\angle s$) $\Rightarrow 2\angle 1 + 2\angle 3 = 180°[using\ (i) and\ (ii)]$
$\Rightarrow \angle 1 + \angle 3 = 90° \ldots\ldots\ldots (iii)$
In $\Delta OAB, \angle AOB + \angle 1 + \angle 3 = 180°\ (sum\ of\ angles\ of\ a\ \Delta) \Rightarrow \angle AOB + 90° = 180°$
$\Rightarrow \angle AOB = 90°\ [using\ (iii)]$

25. Let AB be the tower of height h (units).AP = a, QA = b, as the angles of elevation are
complementary , If $\angle APB = \theta° then \angle AQB = 90° - \theta$.
From right angled ΔBPA, we get $\tan\theta° = \frac{h}{a}$.............(i)
From right angled ΔBQA, we get $\tan(90°-\theta) = \frac{h}{b} \Rightarrow \cot\theta = \frac{h}{b}$ ....(ii)
Multiplying (i) and (ii), we get $\tan\theta° \cot\theta° = \frac{h}{a} \times \frac{h}{b} \Rightarrow 1 = \frac{h^2}{ab} \Rightarrow h^2 = ab \Rightarrow h = \sqrt{ab}$

Hence , the height of the tower is $\sqrt{ab}$ units.

26. Since tangent is perpendicular to radius, OP $\perp$ PT. In ΔOPT ,$\angle$OPT = 90°. By Pythagoras theorem,
$PT^2 = OA^2 - OP^2 = 13^2 - 5^2 = 144 \Rightarrow PT = 12$ cm
AS the lengths of tangent drawn from a point to a circle are equal,
AP = AE = x cm (say) $\Rightarrow$ AT = PT – AP = (12 –x) cm ET = OT – OE = 13 cm – 5 cm = 8 cm.

Given, AB is tangent to the circle at E , OE $\perp$ AB $\Rightarrow \angle AET = 90°$.

In $\Delta AET, \angle AET = 90°$. By Pythagoras theorem, $AT^2 = AE^2 + ET^2 \Rightarrow (12-x)^2 = x^2 + 8^2$

$\Rightarrow 144 - 24x + x^2 = x^2 + 64 \quad \Rightarrow 24x = 80 \Rightarrow x = \frac{10}{3}. \quad \therefore AE = \frac{10}{3}$ cm.

$Similarly, BE = \frac{10}{3}$cm. $\therefore AB = AE + BE = \left(\frac{10}{3} + \frac{10}{3}\right) cm = \frac{20}{3}$ cm.

27. Let AB be a vertical tower and BP be a flagstaff of height h surmounted on the tower AB , and O be the point of observation on the plane (ground) , then BP = h .

Let OA = d and AB = H. Given , $\angle BOA = \alpha \ and \ \angle POA = \beta$.

From right angled Δ OAB , We get $\tan \alpha = \frac{h}{d}$ ……….. (i)

From right angled ΔOAP , we get $\tan \beta = \frac{H+h}{d}$ ……….. (ii)

On dividing (i) by (ii) , we get $\frac{tan\alpha}{tan\beta} = \frac{H}{H+h} \Rightarrow H \tan \alpha + h \tan \alpha = H \tan \beta$

$\Rightarrow h \tan \alpha = H (\tan \beta - \tan \alpha) \quad \Rightarrow H = \frac{htan\,\alpha}{tan\beta - tan\alpha}$, as required .

28. Given vertices of ΔABC as A (a,b), B (b,c) , C (c,a) and the centroid of ΔABC is G (0,0).

As we know that centroid of Δ whose vertices are (x_1, y_1), (X_2, y_2) and (x_3, y_3)

Is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$. $\therefore (0,0) = \left(\frac{a+b+c}{3}, \frac{b+c+a}{3}\right) \Rightarrow a+b+c = 0$ ………….(i)

We Know that $a^3+b^2+c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$\Rightarrow a^3+b^3+c^3 - 3abc = 0 \times (a^2+b^2+c^2-ab-bc-ca)$ (using (i))

$\Rightarrow a^3+b^3+c^3 - 3abc = 0 \quad \Rightarrow a^3+b^3+c^3 - 3abc$.

29. Given, AB = 16 cm , so AL = BL – 8 cm ($\because$ OP is the perpendicular bisector of AB)

In $\Delta OLB , \angle OLB = 90°$ ($\because$ OP$\perp$AB)

BY Pythagoras theorem, $OL^2 = OB^2 - BL^2 = 10^2 - 8^2 = 100 - 64 = 36 \Rightarrow OL = 6$ cm .

Let LP = x cm and BP = y cm , Then OP = OL + LP = (6+x) cm.

Since tangent is perpendicular to radius OB $\perp$PB. In $\Delta OPB, \angle OBP = 90°$. By Pythagoras theorem,

$OP^2 = BP^2 + OB^2 \Rightarrow (x+6)^2 = y^2 + 10^2 \Rightarrow x^2 + 12x + 36 = y^2 + 100 \Rightarrow x^2 - y^2 + 12x = 64$…………(i)

In ΔBLP , $\angle BLP = 90°$($\because OP \perp AB$)

By Pythagoras theorem, $BP^2 = LP^2 + LB^2 \Rightarrow y^2 = x^2 + 8^2 \Rightarrow y^2 = x^2 + 64$ …………….(ii)

Substituting the value of y^2 from (ii) in (i) , we get $x^2 - (x^2 + 64) + 12x = 64$

$\Rightarrow 12x = 128 \Rightarrow x = \frac{32}{3}$ ……………..(iii)

Substituting the value of x from (iii) in (ii) we get $y^2 = \left(\frac{32}{3}\right)^2 + 64 = \frac{1600}{9} \Rightarrow y = \frac{40}{3}$

$\therefore BP = \frac{40}{3}$ cm. $\therefore$ Length of AP $= \frac{40}{3}$ cm ($\because$ AP = BP ,Lengths of tangents)

30. Let AB be the tree and B be the position of bird.

Let AC = x m and AE = y m. In Δ ABC, $\angle A = 90°$ $\tan 45° = \frac{AB}{AC} \Rightarrow 1 = \frac{80}{x} \Rightarrow x$ 80 cm ………(i)

In ΔCDE , $\angle E = 90°$ $\tan 30° = \frac{DE}{CE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{80}{x+y} \Rightarrow x + y = 80\sqrt{3} \Rightarrow y = 80\sqrt{3} - 80$

$\Rightarrow y = 80(\sqrt{3} - 1) = 80(1.732 - 1) \Rightarrow y = 80 \times 0.732 \quad \Rightarrow y = 58.56$ m

Hence , the speed of flying of the bird $= \frac{Distance}{Time} = \frac{ym}{2s} = \frac{58.56}{2}\ m/s$ = 29.28 m/s

Hence , the speed of flying of the bird = 29.28 m/s.