MARKING SCHEME FOR PRACTICE TEST 1St

CLASS - 12TH MATHEMATICS

SECTION - A

1. $-\frac{\pi}{2}$ 1

2. $|adj(adjA)|= 81$ 1

3. $[a\ b\ c] = -30$ 1

4. $\vec{r} = 3\hat{\imath} - 4\hat{\jmath} + 3\hat{k} + \mu(-3\hat{\imath} + 7\hat{\jmath} + 2\hat{k})$ 1

SECTION –B

5. R = $\{(2,8), (3,27)\}$ Range = $\{8,27\}$ 1+1

6. a,b$\in R$ $a * e = a$ 1

$\sqrt{a^2 + e^2}$ = a 1

e = 0

7. f(x)= $(3 - x^3)^{1/3}$ 1+1

Fof(x)= f(f(x))=f($(3 - x^3)^{1/3}$)=x

8. $\tan^{-1}\left[\frac{3a^2x-x^3}{a^3-3ax^2}\right]$ 1

Put x= $a\tan\theta$ 1

= $\tan^{-1}\tan 3\theta = 3\,\theta = 3\tan^{-1}\frac{x}{a}$

9. $\begin{vmatrix} x & y & 1 \\ 1 & 3 & 1 \\ -1 & 7 & 1 \end{vmatrix} = 0$ 1 1

Then equation of line is 2x + y + 5=0

10. Let I represent income I= $\begin{bmatrix} 3x \\ 4x \end{bmatrix}$ and E represent expenditure E= $\begin{bmatrix} 5y \\ 7y \end{bmatrix}$

I-E= S $\begin{bmatrix} 3x \\ 4x \end{bmatrix} - \begin{bmatrix} 5y \\ 7y \end{bmatrix} = \begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$ 1

Then 3x-5y=15000, 4x-7y =15000 1

X=30000 , y= 15000

So monthly income of Aryan and Babbar are 90000,120000

Value – save money will help in odd situation .

11. Equation of plane $\begin{vmatrix} x-0 & y+1 & z+1 \\ 4 & 6 & 4 \\ 3 & 6 & 5 \end{vmatrix} = 0$ 1 1

3x - 4y + 3z - 1= 0

12. $|\hat{a} - \hat{b}|^2 = |\hat{a}|^2 + |\hat{b}|^2 - 2|\hat{a}||\hat{b}|\cos\theta$ 1

$=2(1 - \cos\theta\ )$

$=4\sin^2\frac{\theta}{2}$ 1

$$\sin\frac{\theta}{2} = \frac{1}{2}|\hat{a} - \hat{b}|$$

SECTION- C

13. **Reflexive** – $(a,b)R(a,b)$ 1

Symmetric - (a,b)R(c,d)=(c,d)R(a,b) 1

Transitive –(a,b)R(c,d) and (c,d)R(e,f) = (a,b)R(e,f) 2

14. fog(x)= $\left||x|-3\right|$, gof(x) =$\left||x|\right|-|x|$ 1+1

Then fog(-3)=0, fog(3)=0 ,gof(x)=0 1+1

15. step -1 1

Step-2 1

Step-3 WhereA^2 =O , I^2=I 1+1

16. $\tan^{-1}\left(\frac{5x}{1-x^2}\right)=\frac{\pi}{4}$ 2

X= -1,1/6 but x=1/6 possible only 2

17. $\left(\vec{a}\times\vec{b}\right)^2$ = $|\vec{a}|^2\left|\vec{b}\right|^2\sin^2\theta$ 1

= $|\vec{a}|^2\left|\vec{b}\right|^2-|\vec{a}|^2\left|\vec{b}\right|^2\cos^2\theta$ 1+1

= $|\vec{a}|^2\left|\vec{b}\right|^2-\left(\vec{a}.\vec{b}\right)^2$ 1

18. $\left[\vec{a}+\vec{b}\vec{b}+\vec{c}\vec{c}+\vec{a}\right]=\ (\vec{a}+\vec{b}).\left((\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\right)$ 1

= $(\vec{a}+\vec{b}).\left(\vec{b}\times\vec{c}+\vec{c}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a}\right)$ 2

= $\left[\vec{a}\vec{b}\vec{c}\right]+\left[\vec{b}\vec{c}\vec{a}\right]$ 1

= $2\left[\vec{a}\vec{b}\vec{c}\right]$

18(OR)

Let $\vec{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$ if $|\vec{a}|=1$ 1

$a_1{}^2+a_2{}^2+a_3{}^2=1$ 1

$\vec{a}.\hat{\imath}=\frac{1}{\sqrt{2}}$,$\vec{a}.\hat{\jmath}=\frac{1}{2}$,$\vec{a}.\hat{k}=\cos\theta$ 1

Then $\cos\theta=\frac{1}{2}$ 1

19. Let a,b,c are direction ratio of normal to the required plane

a$(x+1)+b(y-3)+c(z-2)=0$ ....(1) 1

which is perpendicular top x + 2y+ 3z= 5 , 3x + 3y+ 2z = 8 1

then a + 2b + 3c = 0, 3a + 3b +2c =0

$\frac{a}{-5}=\frac{b}{7}=\frac{c}{-3}$ = k then a=-5k , b=7k , c=-3k put the value of a, b, c 1

In equation 1 then required equation of plane is 5x + 7y- 3z- 10 = 0 1

20. Required equation of plane is

(2x + y –z -3)+$\mu(5x-3y+4z+9)$ =0 ....(1) 1

x (2+5μ) + $y(1-3\mu)+z(-1+4\mu)+9\mu-3$ =0 1

is parallel to line with direction ratios are 2,4, 5

then normal to the plane are perpendicular to line 1

then 2(2+5μ)+4(1-3μ)+5(-1+4μ)=0 1

18μ − 3 =0 then $\mu=1/6$ put in eqn 1

Then required equation of plane is 17x+3y-2z-9=0

20(OR)

Equation of plane $\begin{vmatrix} x & y+1 & z \\ 1 & 2 & 1 \\ 3 & 4 & 0 \end{vmatrix}$ =0

4x-3y+2z-3=0

Vector form $\vec{r}.\left(4\hat{\imath}+3\hat{\jmath}+2\hat{k}\right)=3$

Normal equation of plane $\vec{r}\left(\frac{4\hat{\imath}-3\hat{\jmath}+2\hat{k}}{\sqrt{29}}\right)=\frac{3}{\sqrt{29}}$

21. applying$c_1 \to c_1+c_2+c_3$

$=\begin{vmatrix} x+y+z & y-x & z-x \\ x+y+z & 3y & z-y \\ x+y+z & y-z & 3z \end{vmatrix}$

Taking common x +y +z from c_1

= (x+y+z) $\begin{vmatrix} 1 & y-x & z-x \\ 1 & 3y & z-y \\ 1 & y-z & 3z \end{vmatrix}$

= Applying$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

= (X+Y+Z) $\begin{vmatrix} 1 & y-x & z-x \\ 0 & 2y+a & x-y \\ 0 & x-z & 2z+x \end{vmatrix}$

= 3(x+y+z)(xy+yz+zx)

22.

If A = $\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$

Then $|A|=1+\tan^2 x$

AdjA= $\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$ 2

Then $A^T A^{-1}=\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$ 2

23. f(t) = $\begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t^2 & 2t \\ \sin t & t & 1 \end{vmatrix}$

$R_1 \to R_1 - R_3, R_2 \to R_2 - R_3$

f(t)= $\begin{vmatrix} \cos t - \sin t & 0 & 0 \\ \sin t & x^2 - x & 2t-1 \\ \sin t & t & 1 \end{vmatrix}$

f(t)= $(\sin t - \cos t)t^2$

$\lim_{x\to 0}\frac{f(t)}{t^2}=\lim_{x\to 0}(\sin t - \cos t)$ =-1

SECTION - D

24.

AB = $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

AB=I

$A^{-1}(AB)=A^{-1}I$ 1

B=A^{-1}

x-y+2z=1 ,0x+ 2y - 3z = 1 , 3x-2y + 4z = 2 2

X=$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ A=$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ -3 & -2 & 4 \end{bmatrix}$ B=$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ 1+1

X=$A^{-1}B$ x=0, y=5, z=3

25. $(a,b)*(c,d) = (ab+bc, bd)$ 1

1) Commutative (a,b)$*(c,d) = (c,d)*(a,b)$ 1

2) Assóciative (a,b)$*[(c,d)*(e,f)]$ =$[(a,b)*(c,d)]*(e,f)$ 2

3) Identity (a,b)$*(x,y) = (a,b)$ 2

(ay+bx, by)= (a,b) 1

x=0, y=1 e=(0,1) $does\ not\ belong\ N \times N$

26. $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$

applying$c_1 \to c_1 - bc_3$, $c_2 \to c_2 + ac_3$ 1

=$\begin{bmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b(1+a^2+b^2) & -a(1+a^2+b^2) & 1-a^2-b^2 \end{bmatrix}$ 1

taking common (1 +$a^2 + b^2$) 2

= $(1+a^2+b^2)^2 \begin{bmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{bmatrix}$ 2

= $(1+a^2+b^2)^3$

OR

$$\begin{bmatrix} (b+c)^2 & ba & ca \\ ab & (c+a)^2 & cb \\ ac & bc & (a+b)^2 \end{bmatrix}$$

multiplya,b,c in R_1, R_2, R_3 2

=$\frac{1}{abc}\begin{vmatrix} a(b+c)^2 & a^2b & a^2c \\ ab^2 & b(c+a)^2 & cb^2 \\ ac^2 & bc^2 & c(a+b)^2 \end{vmatrix}$

taking common a,b,c from R_1, R_2, R_3 2

applying$c_1 \to c_1 - c_2,\ c_2 \to c_2 - c_3$

=$(a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ b-c-a & c+a-b & b^2 \\ 0 & c-a-b & (a+b)^2 \end{vmatrix}$ 2

after applying $R_3 \to R_3 - R_1 - R_2$ and $c_1 \to ac_1 + c_3$,$c_2 \to bc_2 + c_3$

=2abc$(a+b+c)^3$.

27. A= $\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

then co-factors of A is C= $\begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}$ 2

adjA = C^T= $\begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$ 1+1

AadjA= $\begin{bmatrix} 25 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 25 \end{bmatrix}$ =25 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ = $|A|I$ 2

28. $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = r \quad and \quad \frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu$ 1

if point P on line 1 then P(3r-1,5r-3,7r-5) and Q($\mu + 2, 3\mu + 4, 5\mu + 6$) 1+1

if lines are intersecting the P and Q are same so 1+1

3r-$\mu = 3$,$5r - 3\mu = 7$ then r =$\frac{1}{2}$, $\mu = -\frac{3}{2}$

points are same hence line will intersect then point of intersection ($\frac{1}{2}$,$\frac{-1}{2}$,$\frac{-3}{2}$) . 1

29. P =(-2,3,4)

$\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ =r 1

then Q(3r-2 ,$\frac{4r-3}{2}$, $\frac{5r-4}{3}$) then direction ratio of PQ are (3r ,$\frac{4r-9}{2}$,$\frac{5r+8}{3}$) 1+1

4x+12y-3z+1 = 0 1

PQ Is parallel to plane then normal is perpendicular to Dr of line 1

4(3r)+12($\frac{4r-9}{2}$) -3$\left(\frac{5r+8}{3}\right)$=0 r=2 1

then point Q = (4,5/2,2) PQ= $\frac{17}{2}$

OR l + m + n=0 , 3 lm- 5mn+ 2nl=0 1

let$l_1, m_1, n_1 and\ l_2, m_2, n_2$ are direction cosine 1+1

3lm+5m(l+m)-2i(l+m)=0

3lm + 5lm + $5m^2 - 2l^2 - 2lm$=0 1

$2\left(\frac{l}{m}\right)^2 - 6\frac{l}{m} - 5 = 0$ 1+1

$\left(\frac{l_1}{m_1}\right)\left(\frac{l_2}{m_2}\right) = \frac{-5}{2} \frac{l_1 l_2}{-5} = \frac{m_1 m_2}{2}$ similarly $\frac{l_1 l_2}{-5} = \frac{m_1 m_2}{3}$

$l_1 l_2 + m_1 m_2 + n_1 n_2 = -5k + 2k + 3k = 0$