D.A.V PUBLIC SCHOOL, HEHAL, RANCHI

Std – X | Practice Test – I | Maths | F.M. – 80

Marking Scheme

Section – A

1. 5 x(7x6x4x3x2x1 + 1) = 5 x (1008 + 1)
   = 5 x 1009, which is product of primes. Hence, it is composite number. 1
2. $\alpha + \beta = -\frac{b}{a} = -\frac{7}{2}$ and $\alpha.\beta = \frac{c}{a} = \frac{5}{2}$
   $\therefore \alpha + \beta + \alpha.\beta = -7/2 + 5/2 = -2/2 = -1$ 1
3. $\because$ The lines are intersecting, hence, unique solution exists.
   $\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow 3/a \neq 2/-1 \Rightarrow 2a \neq -3$
   $\therefore a \neq -3/2$ 1
4. $D = b^2 - 4ac$ (a=4, b=4√3, c=3)
   $=(4\sqrt{3})^2 - 4x4x3 \Rightarrow 16x3 - 48 = 48 - 48 = 0$.
   $\because D = 0$
   $\therefore$ Roots are real and equal. 1
5. $\because$ Roots are equal
   $\therefore D = 0$ or, $b^2 - 4ac = 0$
   $\Rightarrow (8k)^2 - 4x9x16 = 0$
   $\Rightarrow 64k^2 - 576 = 0 \Rightarrow 64k^2 = 576$
   or, $k^2 = 576/64 = 9$
   $\therefore k = \pm 3$. 1
6. $\because$ k, 2k – 1, 2k + 1 are in AP
   $\therefore 2k - 1 - k = 2k + 1 - 2k + 1$
   $\Rightarrow k - 1 = 2 \therefore k = 3$. 1

Section – B

7. Given, HCF(253, 440) = 11 and LCM(253, 440) = 253xR 1
   $\therefore LCM(253, 440) = \frac{253 \times 440}{HCF(253,440)} \Rightarrow 253 \times R = \frac{253 \times 440}{11} \Rightarrow R = \frac{253x440}{253x11} \therefore R = 40$ 1
8. No, because $6^n = (2x3)^n$ 1
   If the number 6^n ends with digit 5, then it will be divisible by 5,i.e., its one factor will be 5. But the only primes in the factorization of 6^n are 2 and 3, but not 5.
   Hence, it cannot ends with digit 5. 1
9. Given sum of zeroes = √2 i.e. $\alpha + \beta = \sqrt{2}$ and product of zeroes = $-\frac{3}{2}$ i.e. $\alpha.\beta = -3/2$ 1
   Required polynomial is $x^2 - (\alpha + \beta)x + \alpha.\beta$
   $\Rightarrow x^2 - \sqrt{2}x - 3/2$ or $½(2x^2 - 2\sqrt{2}x - 3)$ 1
10. For infinitely many solutions
    $a_1/a_2 = b_1/b_2 = c_1/c_2 \Rightarrow \frac{2}{a+b} = \frac{3}{2a-b} = \frac{7}{21}$
    $\Rightarrow \frac{2}{a+b} = \frac{1}{3} \Rightarrow a+b = 6$ --------(i)
    And $\frac{3}{2a-b} = \frac{1}{3} \Rightarrow 2a - b = 9$ --------(ii) 1
    Adding (i) and (ii), 3a = 15 $\therefore$ a= 5
    Putting the value of a in eq (i), $\therefore$ b = 1 1
11. Given quadratic equation is
    $a^2b^2x^2 + b^2x - a^2x - 1 = 0$
    or, $b^2x(a^2x + 1) - 1(a^2x + 1) = 0$
    or, $(a^2x + 1)(b^2x - 1) = 0$ 1
    or, $a^2x + 1 = 0$ and $b^2x - 1 = 0$
    $\Rightarrow x = -1/a^2$ or, $x = 1/b^2$ 1
12. All three digit natural numbers, multiple of 11 are

110, 121, 132 990

Here, common difference is 11and 1st term is 110, $a_n(l) = 990$

$a_n = a + (n-1)d$

or, $990 = 110 + (n-1)11$

or, $880 = (n-1)11$

or, $n = \frac{880}{11} + 1 \Rightarrow n = 81$ 1

now, $S_n = \frac{n}{2}[a + l]$

or, $S_{81} = \frac{81}{2}[110 + 990]$

or, $S_{81} = 81/2 \times 1100 = 81 \times 550$

$\therefore S_{81} = 44550.$ 1

Section – C

13. Let $5 - 2\sqrt{3}$ be a rational no.

Now $5 - 2\sqrt{3} = \frac{a}{b}$ (a, b are integers and $b \neq 0$) 1

$\Rightarrow 5 - \frac{a}{b} = 2\sqrt{3} \Rightarrow \frac{5b-a}{2b} = \sqrt{3}$

$\because$ a, b are integers $\therefore \frac{5b-a}{2b}$ is a rational number and $\sqrt{5}$ is an irrational number. 1

Here, a contradiction arises, that rational number = irrational number

This shows that our assumption is wrong. So, $5 - 2\sqrt{3}$ is irrational. 1

14. Given $x = 0.56125$

$\therefore x = \frac{56125}{100000} = \frac{449}{800}$ 1

Here $q = 800$ or $q = 2^5 \times 5^2$ 1

$\Rightarrow 2^n \times 5^m = 2^5 \times 5^2 \therefore n = 5$ and $m = 2$ 1

15. Given α and β are the zeroes of the polynomial

$p(x) = ax^2 + bx + c$

$\therefore$ Sum of zeroes, $\alpha + \beta = \frac{-b}{a}$ and product of zeroes, $\alpha\beta = \frac{c}{a}$ 1

Now, $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta)$

$= \frac{c}{a} \times \frac{(-b)}{a} = \frac{-bc}{a^2}$ 2

16. Let p(x) be zero

$\Rightarrow 4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$\Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$ 1

$\Rightarrow 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$

$\Rightarrow (4x - \sqrt{3})(\sqrt{3}x + 2) = 0$

$\Rightarrow 4x - \sqrt{3} = 0$ or, $\sqrt{3}x + 2 = 0$

$\Rightarrow x = \frac{\sqrt{3}}{4}$ or, $x = \frac{-2}{\sqrt{3}}$ 1

Sum of zeroes $= \frac{\sqrt{3}}{4} - \frac{2}{\sqrt{3}} = \frac{3-8}{4\sqrt{3}} = \frac{-5}{4\sqrt{3}} = \frac{-\text{ coefficient of } x}{\text{coefficient of } x^2}$

Product of zeroes $= \frac{\sqrt{3}}{4} \times \frac{-2}{\sqrt{3}} = \frac{2\sqrt{3}}{4\sqrt{3}} = \frac{\text{coefficient term}}{coefficient\ of\ x^2}$ 1

17. Let the two numbers be x and y(x>y)

According to the equation

$x - y = 26$ -------(i) 1

and $x = 3y$ ----------(ii)

on putting the value of x from eq (ii) in eq(i)

we get $3y - y = 26$

$\Rightarrow 2y = 26 \Rightarrow y = 13$ 1

On substituting y=13 in eq (ii), we get

$x = 3 \times 13 \Rightarrow x = 39$ 1

$\therefore$ the two numbers are 39 and 13

or,

let the required numbers be x and y, where x>y

Given, $x^2 - y^2 = 180$ ------(i)

And, $y^2 = 8x$ --------(ii) 1

From eq (i) and (ii) we get

$X^2 - 8x = 180 \Rightarrow x^2 - 8x - 180 = 0$

$\Rightarrow x^2 - 18x + 10x - 180 = 0$

$\Rightarrow x(x - 18) + 10(x - 18) = 0$

$\Rightarrow (x - 18)(x + 10) = 0$

$\Rightarrow x - 18 = 0$ or, $x + 10 = 0$

$\Rightarrow x = 18$ or, $x = -10$ 1

Now, if x = 18, then square of smaller number = 8 x 18 =144

$\Rightarrow$ Smaller number = ±12 $\Rightarrow$ Smaller number = 12 or -12

And if x = -10, then square of smaller number = 8 x (-10) = -80, which is not possible as square of number cannot be negative.

Hence, the required numbers are 18 and 12 or 18 and - 12. 1

18. Yes, let us divide

$2t^4 + 3t^3 - 2t^2 - 9t - 12$ by $t^2 - 3$

The division process is

$$\begin{array}{r|l} & 2t^2 + 3t + 4 \\ t^2 - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\ & 2t^4 \quad\quad - 6t^2 \\ & - \quad\quad\quad + \\ \hline & 3t^3 + 4t^2 - 9t - 12 \\ & 3t^3 \quad\quad - 9t \\ & - \quad\quad\quad + \\ \hline & 4t^2 - 12 \\ & 4t^2 - 12 \\ & - \quad + \\ \hline & 0 \end{array}$$

1

1

Here, the remainder is 0, therefore $t^2 - 3$ is a factor. 1

19. $\frac{x}{a} + \frac{y}{b} = a + b$ -------(i)

$\frac{x}{a^2} + \frac{y}{b^2} = 2$ ------(ii)

Multiplying eq (i) by 1/a and subtracting eq (i) and (ii) 1

$\frac{x}{a^2} + \frac{y}{ab} = 1 + \frac{b}{a}$

$\frac{x}{a^2} + \frac{y}{b^2} = 2$

$-\quad -\quad -$

$\frac{y}{b}(\frac{1}{a} - \frac{1}{b}) = \frac{b}{a} - 1$

$\Rightarrow \frac{y}{b}(\frac{b-a}{ab}) = \frac{b-a}{a}$

$\Rightarrow \frac{y}{b} = \frac{b-a}{a} \times \frac{ab}{b-a} \therefore y = b^2$ 1

Putting the value of y in eq (ii)

$\frac{x}{a^2} + \frac{y}{b^2} = 2 \Rightarrow \frac{x}{a^2} + \frac{b^2}{b^2} = 2$

$\Rightarrow \frac{x}{a^2} = 2 - 1 \Rightarrow \frac{x}{a^2} = 1 \Rightarrow x = a^2$

$\therefore x = a^2$ and $y = b^2$ 1

20. Let the age of one of the friend be x yr.

Then, age of other friend = (20 – x)yr [∵ the sum of the ages of two friends is 20 yrs] 1

4 yr ago, age of one of two friends = (x – 4)yr

And age of the other friend = (20 – x – 4)yr = (16 – x)yr

According to the question,

$(x - 4)(16 - x) = 48$

or, $16x - x^2 - 64 + 4x = 48$

or, $x^2 - 20x + 112 = 0$ 1

on comparing with $ax^2 + bx + c = 0$, we get

a = 1, b = - 20 and c = 112

Now, discriminant, $D = b^2 - 4ac = (-20)^2 - 4 \times 1 \times 112$

$= 400 - 448 = -48 < 0$

which implies that the real roots are not possible because this condition represents noreal roots. So, the solution does not exist and hence given situation is not possible. 1

21. Given AP is 121, 117, 113, ...

Here, first term, a =121 and common difference, d = 117 – 121 = - 4 1

Let *nth* term of this AP be the first negative term. Then, $a_n < 0$

$\Rightarrow a + (n-1)d < 0 \Rightarrow 121 + (n-1)(-4) < 0$ 1

$\Rightarrow 125 - 4n < 0 \Rightarrow 125 < 4n$ or, $4n > 125$

$\Rightarrow n > 125/4 \Rightarrow n > 31\frac{1}{4}$

Least integral value of n is 32

Hence, 32^{nd} term of the given AP is the first negative term 1

22. $a = (-5)$, $a_n(l) = -230$ and $d = (-8) - (-5) = -3$

$a_n = a + (n-1)d \Rightarrow -230 = -5 + (n-1)(-3)$ 1

$\Rightarrow -225 = (n-1) \times (-3) \Rightarrow n - 1 = 75 \quad \therefore n = 76$ 1

$S_n = \frac{n}{2}(a + l) \Rightarrow S_{76} = \frac{76}{2}(-5 - 230) = 38 \times (-225) = -8930$ 1

Section - D

23. Let *a* be any odd positive integer, then on dividing *a* by *b*, we have

$a = bq + r$, $0 \le r < b$ ----(i) [by Euclid's division lemma] 1

on putting b = 2 in eq (i), we get

$a = 2q + r$, $0 \le r < 2 \Rightarrow r = 0$ or 1

if r =0, then a = 2q, which is divisible by 2. So, 2q is even 1

if r = 1, then a = 2q + 1, which is not divisible by 2.

$\therefore (2q + 1)$ is odd. 1

Now as *a* is odd, so it cannot be of the form 2q. Thus, any odd positive integer *a* is of the form (2q + 1).

Now consider $a^2 = (2q+1)^2 = 4q^2 + 1 + 4q$

$= 4(q^2 + q) + 1 = 4m + 1$ where $m = q^2 + q$

Hence, for some integer *m*, the square of any positive integer is of the form 4m + 1 1

24. The HCF of 420 and 130.

Here, 420 > 130 1

Now, $420 = 130 \times 3 + 30$, $130 = 30 \times 4 + 10$ } by Euclid's division lemma

$30 = 10 \times 3 + 0$ 2

$\therefore$ Required number of burfis = 10 1

25. Given, two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

So, $\left(x - \sqrt{\frac{5}{3}}\right)$ and $\left(x + \sqrt{\frac{5}{3}}\right)$ are the factors of given polynomial. 1

$\Rightarrow \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = x^2 - \frac{5}{3} = \frac{3x^2 - 5}{3}$ is a factor of given polynomial.

Consequently, $3x^2 - 5$ is a factor of the given polynomial.

Now, let us divide $3x^4 + 6x^3 - 2x^2 - 10x - 5$ by $3x^2 - 5$ 1

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
3x^2 - 5 & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & 3x^4 \qquad - 5x^2 \\
 & - \qquad + \\
\hline
 & 6x^3 + 3x^2 - 10x - 5 \\
 & 6x^3 \qquad - 10x \\
 & - \qquad + \\
\hline
 & 3x^2 - 5 \\
 & 3x^2 - 5 \\
 & - \quad + \\
\hline
 & 0
\end{array}
$$

Here, quotient = $x^2 + 2x + 1 = (x+1)^2$ and the zeroes of $(x+1)^2$ are -1 and -1. Hence, all zeroes of given polynomial are $\sqrt{5/3}$, $-\sqrt{5/3}$, - 1 and -1. 2

26. Let speed of boat in still water be x km/hr

And that of stream be y km/hr

Speed of boat in downstream = (x + y) km/hr

Speed of boat in upstream =(x – y) km/hr 1

ATQ, $\frac{40}{x+y} + \frac{12}{x-y} = 8$ -----(i) x 4

And, $\frac{32}{x+y} + \frac{16}{x-y} = 8$ -----(ii) x 3 1

$\frac{160}{x+y} + \frac{48}{x-y} = 32$

$\frac{96}{x+y} + \frac{48}{x-y} = 24$

- - -

$\frac{64}{x+y} = 8 \therefore x + y = 8$ -----(iii)

Putting the value of x + y in eq (i)

$\frac{40}{8} + \frac{12}{x-y} = 8 \Rightarrow x - y = 4$ ----(iv) 1

On adding eq (iii) and (iv), we get

x = 6 and y = 2

∴ Speed of boat in still water = 6 km/hr

And speed of stream = 2 km/hr 1

27. (i) Let the usual speed of the aeroplane be x km/hr.

Then, the time taken to cover 1200 km = 1200/xhr

Now, if the plane started late by one hour, then its speed = (x +100) km/hr and time taken to cover 1200 km = $\frac{1200}{x+100}$hr1

ATQ, $\frac{1200}{x} - \frac{1200}{x+100} = 1$

Or, $1200\left[\frac{1}{x} - \frac{1}{x+100}\right] = 1$

Or, $1200\left[\frac{x+100-x}{x(x+100)}\right] = 1$

Or, $120000 = x^2 + 100x$

Or, $x^2 + 100x - 120000 = 0$ 1

Or, $x^2 + 400x - 300x - 120000 = 0$

Or, $x(x+400) - 300(x+400) = 0$

Or, $(x+400)(x-300) = 0$

⇒ x = -400(neglect) or x = 300

∴ usual speed of the aeroplane is 300 km/hr. 1

(ii) The values(qualities) of the pilot represented in this question are leadership and punctuality. 1

28. We have, $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

$\Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{1}{a+b+x} - \frac{1}{x}$ 1

$\Rightarrow \frac{a+b}{ab} = \frac{x-a-b-x}{(a+b+x)x}$

$\Rightarrow \frac{a+b}{ab} = \frac{-(a+b)}{(a+b+x)x}$ 1

$\Rightarrow \frac{1}{ab} = \frac{-1}{(a+b+x)x}$

$\Rightarrow x^2 + ax + bx + ab = 0$ 1

$\Rightarrow x(x + a) + b(x + a) = 0$

Or, $(x + b)(x + a) = 0$

∴ x = -a or, x = -b 1

29. Let a and d be the first term and common difference of the given AP

Given, $a_4 = 0$

⇒ a + 3d = 0 ⇒ a = -3d ----(i) 1

To prove: $a_{25} = 3a_{11}$

Proof: $a_{25} = a + 24d$

Or, $a_{25} = -3d + 24d$ 1

Or, $a_{25} = 21d$ -----(ii)

Now, $a_{11} = a + 10d$

Or, $a_{11} = -3d + 10d$

Or, $a_{11} = 7d$ 1

Or, $3a_{11} = 21d$ ------(iii)

From eq (ii) and (iii), we get

$a_{25} = 3a_{11}$ ***Proved*** *1*

30. Let a be the first term and d be the common difference of given AP

Given $a_5 + a_9 = 72$

$\Rightarrow a + 4d + a + 8d = 72 \Rightarrow 2a + 12d = 72$ -----(i) 1

Also, $a_7 + a_{12} = 97 \Rightarrow a + 6d + a + 11d = 97 \Rightarrow 2a + 17d = 97$ ----(ii) 1

On subtracting eq (i) and (ii), we get

$-5d = -25 \Rightarrow d = 5$ 1

Putting the value of d in eq (i), we get

$a = 6$

$\therefore$ Required AP is a, a+d, a+2d,.............

$\therefore$ 6, 11, 16, ... 1
